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Domain Walls Strange Quark Matter in Einstein-Rosen Space-Time with Cosmological Constant and Heat Flow

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Abstract In this paper, we have studied the solutions of Einstein field equations for domain walls with cosmological constant and heat flow in Einstein-Rosen cylindrical symmetric space-time when strange quark matter and normal matter attached to the domain walls. Some physical and kinematical features of the obtained cosmological model are studied and discussed.

Keywords Strange quark matter · Domain wall · Cylindrical symmetric space-time · Cosmological constant and heat flow

1 Introduction

In recent years, symmetry is proving to be a powerful unifying tool in particle physics and cosmology because through symmetry and symmetry breaking, particles which appear to be different in mass, charge etc. can be understood as different states of a single unified field theory in which all particles and fundamental forces of nature to unify gravity are related through a broken symmetry. Certain grand unified field theory predicts topological defects which might have been formed in the early phase transition of the universe. These defects are stable field configurations which arise in field theories with spontaneously broken discrete symmetries. Spontaneous symmetry breaking is an old idea, described within the particle physics in terms of the Higgs-Kibble field mechanism [16]. The symmetry is spontaneously broken because the ground state is not invariant under the full symmetry of the Lagrangian

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identities. Thus the expected value of Higgs field in vacuum is non-zero. In quantum field theories, broken symmetries are restored at sufficiently high temperature. The well-known topological defects are domain walls which occur when a discrete symmetry is broken at a phase transition, and the defect density is related to the domain size.

One of the interesting consequences of phase transition in the early universe is the formation of strange quark matter. Itach [15], Bodmer [5] and Witten [24] proposed two ways of formation of quark matter, namely, the quark hadron phase transition in the early universe and conversion of neutron stars in to strange at ultrahigh densities. In this respect, Alcock et al. [1], Haensel et al. [14], Yilmaz [26, 27] and Yavuz et al. [25] has been confined his work to the quark matters which attached to the topological defects in general relativity. Typically, the strange quark matter is modeled with an equation of state based on the phenomenological bag model of quark matter [11]. In this model, quark matters are degenerate Fermi gases which exists only in a region of space endowed with a vacuum energy density B_c (called as the bag constant). Also, the quark matter is composed of mass less u, d quarks, massive s quark and electrons. In the simplified version of the bag model, assuming quarks are mass less and non-interacting, we then have quark pressure $p_q = \frac{1}{3}\rho_q$ (ρ_q is the quark energy density). The total energy density

$$\rho_m = \rho_q + B_c. \tag{1}$$

But total pressure is

$$p_m = p_q - B_c. \tag{2}$$

The equation of state for strange quark matter is

$$p_m = \frac{1}{3}(\rho_m - 4B_c). \tag{3}$$

The equation of state for perfect fluid (Normal matter) is given by

$$p_m = (\gamma - 1)\rho_m, \tag{4}$$

where $1 \le \gamma \le 2$ is a constant.

In general relativistic quantum field theory, the cosmological constant is explained as the vacuum energy density [12, 28, 29]. Negative pressure is a property of vacuum energy, but the exact nature of dark energy remains one of the great mysteries of the big-bang. The basic role of the cosmological constant is related to the observational evidence of high redshift Type Ia supernovae [21, 22] for a small decreasing values of cosmological constant (Λ presence $\le 10^{-56}$ cm^{-2}) at the present epoch. Bergmann [3] have studied the cosmological constant in terms of the Higgs scalar field. Linde [17] proposed that the term Λ is a function of temperature and is related to the process of broken symmetries. Godel [13] have studied cosmological model generated by a solution of the modified Einstein field equations in which the cosmological repulsive term (Λg_{ik}) has been added. Yilmaz [26] proposed the study of domain wall solutions in the non-static and stationary Godel universe with a cosmological constant.

Bergmann [4] proposed the study of the cosmological solutions with heat flux. Heat conduction ensures entropy production on general thermodynamic grounds. In the early stages of the evolution of the universe, matter is supposed to be highly dense and hot, and hence consideration of heat flow in the universe is quite appropriate. The effect of heat flow in the universe has been studied by several authors such as Deng [9], Mukherjee [18], Novello and Rebaucas [19], Bradley and Sviestins [6], Banerjee and Sanyal [2], Coley and Dunn [7], Collins et al. [8] and Singh [23].

In the present paper, we will study the strange quark matter and normal matter attach to the domain walls in the Einstein-Rosen cylindrical symmetric space-time with cosmological constant and heat flow. The paper is organized as follows: In Sect. 2, we have solved the Einstein field equations for strange quark matter and normal matter attached to the domain wall in the context of cylindrical symmetric space-time. In Sect. 3, we have studied the physical and kinematical features and in Sect. 4, discussions are given.

2 Domain wall Solutions in the Einstein-Rosen Space-Time

We consider the Einstein-Rosen cylindrical symmetric line element in the form

$$ds^2 = e^{2\alpha-2\beta}(dt^2 - dr^2) - r^2 e^{-2\beta} d\varphi^2 - e^{2\beta} dz^2, \tag{5}$$

where α and β are functions of t alone. (x^1, x^2, x^3, x^4) corresponds to (r, φ, z, t).

The non-vanishing components of Ricci tensor R_{ik} for the metric (5) are as follows :

$$R_{11} = -\ddot{\alpha} + \ddot{\beta}, \tag{6a}$$

$$R_{22} = r^2 e^{-2\alpha}\ddot{\beta}, \tag{6b}$$

$$R_{33} = -r^2 e^{2\beta} R_{22}, \tag{6c}$$

$$R_{44} = -R_{11} + 2\dot{\beta}^2 \tag{6d}$$

and

$$R_{14} = R_{41} = -\frac{\dot{\alpha}}{r}, \tag{6e}$$

where, overhead dot denotes the differentiation with respect to t.

The energy-momentum tensor of a domain wall with heat flow in the conventional form [20] is given by

$$T_{ik} = (\rho + p)u_i u_k - p g_{ik} + Q_i u_k + Q_k u_i \tag{7}$$

together with the conditions

$$u^i u_i = 1 \quad \text{and} \quad u^i Q_i = 0, \tag{8}$$

where u_i is the four velocity and Q_i is the heat flow vector.

The energy-momentum tensor of the domain wall includes strange quark matter (described by $\rho_m = \rho_q + B_c$ and $p_m = p_q - B_c$) and normal matter (described by ρ_m and p_m) as well as domain wall tensor σ_ω i.e.

$$\rho = \rho_m + \sigma_\omega \quad \text{and} \quad p = p_m - \sigma_\omega. \tag{9}$$

Also p_m and ρ_m are related by the bag model equation of state i.e. (3) and the equation of state i.e. (4).

In co-moving coordinate system, we have $u_a = \delta^i_a u_i$ and $Q_a = \delta^i_a Q_i$. We assume that heat flow is in r-direction. Therefore

$$u_a = (0, 0, 0, e^{\alpha-\beta}) \quad \text{and} \quad Q_a = (Q, 0, 0, 0), \tag{10}$$

where Q is a function of t to be determined from field equations.

The Einstein field equations are

$$R_{ik} = -\left(T_{ik} - \frac{1}{2}T g_{ik}\right) + \Lambda g_{ik}, \tag{11}$$

where $T = g^{ik}T_{ik}$. We use geometrized units so that $8\pi G = c = 1$. Thus from (7), (8), (10) and (11), we obtain

$$R_{11} = -e^{2\alpha-2\beta}\left[\frac{1}{2}(\rho - p) + \Lambda\right], \tag{12a}$$

$$R_{22} = -r^2 e^{-2\beta}\left[\frac{1}{2}(\rho - p) + \Lambda\right], \tag{12b}$$

$$R_{33} = -e^{2\beta}\left[\frac{1}{2}(\rho - p) + \Lambda\right], \tag{12c}$$

$$R_{44} = -e^{2\alpha-2\beta}\left[\frac{1}{2}(\rho + 3p) - \Lambda\right] \tag{12d}$$

and

$$R_{14} = R_{41} = -Qe^{\alpha-\beta}. \tag{12e}$$

The kinematical quantities are given as follows:

The scalar of expansion (θ) is given by

$$\theta = (\dot{\alpha} - \dot{\beta})e^{\beta-\alpha}. \tag{13}$$

The shear scalar (σ) is determined by

$$\sigma^2 = \frac{1}{2}\sigma_{ik}\sigma^{ik} = \frac{2}{3}\theta^2. \tag{14}$$

The proper volume is defined by

$$V^3 = (-g)^{\frac{1}{2}} = re^{2\alpha-2\beta}. \tag{15}$$

The equations (6a)–(6d) and (12a)–(12d) admit an exact solutions as

$$\alpha = at + b \quad \text{and} \quad \beta = ct + d, \tag{16}$$

where a, b, c and d are constants of integration.

Thus the Einstein-Rosen cylindrical symmetric space-time geometry for quark matter coupled to the domain wall (after suitable choice of coordinates and constants of integration) is as follows

$$ds^2 = e^{2t}(dt^2 - dr^2) - r^2e^{-2t}d\varphi^2 - e^{2t}dz^2 \tag{17}$$

and the solution of heat flow conduction is given by

$$Q = \frac{2}{r}e^{-t}. \tag{18}$$

3 The Physical and Kinematical Features

The expressions for physical quantities like pressure (p) and energy density (ρ) for model (17) are as follows:

$$p = p_m - \sigma_\omega = e^{-2t} + \Lambda \tag{19}$$

and

$$\rho = \rho_m + \sigma_\omega = e^{-2t} - \Lambda. \tag{20}$$

For kinematical quantities, from (13)–(15), we obtain

$$\text{Scalar of expansion} \quad \theta = e^{-t},$$
$$\text{Shear scalar} \quad \sigma^2 = \frac{2}{3}e^{-2t} \text{ and}$$
$$\text{proper volume} \quad V^3 = re^{2t}.$$

To determine exactly tension of domain wall i.e. σ_ω and also density and pressure of the quark matter, we will use the equations of state given by (3) and (4) for this, we have the following two cases:

Case (i): The strange quark matter attached to the domain wall:
If we use (3) into (19) and (20), we get

$$\rho_q = \frac{3}{2}e^{-2t}, \tag{21}$$

$$p_q = \frac{1}{3}\rho_q = \frac{1}{2}e^{-2t} \tag{22}$$

and

$$\sigma_\omega = -\frac{1}{2}e^{-2t} - B_c - \Lambda. \tag{23}$$

Case (ii): Normal matter attached to the domain wall:
If we use (4) into (19) and (20), we obtain

$$\rho_m = \rho_q + B_c = \frac{2}{\gamma}e^{-2t}, \tag{24}$$

$$p_m = p_q - B_c = 2\left(\frac{\gamma - 1}{\gamma}\right)e^{-2t} \tag{25}$$

and

$$\sigma_\omega = \left(\frac{\gamma - 2}{\gamma}\right)e^{-2t} - \Lambda. \tag{26}$$

4 Discussion

In this paper, we have considered the solutions of Einstein field equations in Einstein-Rosen cylindrical symmetric space-time with cosmological constant and heat flow when the strange quark matter and normal matter attached to the domain walls. The model (17) has no initial singularity at $t=0$. For model (17), it is observed that the physical and kinematical parameters like p, ρ, σ, θ and V are all constants at $t=0$. As t increases, the scalar of expansion (θ) and shear scalar (σ) decreases, and at the final state of evolution as $t \to \infty$, they vanish (i.e. $\theta = \sigma = 0$). But at large value of t, proper volume diverges. At $t=0$, the heat flow vector is proportional to r-direction and it disappears when $t \to \infty$. At large value of t, the energy density (ρ) and pressure (p) are related by the equation of state $p = -\rho (= \Lambda)$.

It is observe that

$$\frac{\sigma}{\theta} = \sqrt{\frac{2}{3}} \cong 0.82$$

for the model (17). The present upper limit of $\frac{\sigma}{\theta}$ is considered greater than its present value. This fact indicates that our solutions represent the early stages of evolution of the universe.

In cosmology, Baryon conservation law (conservation of total particle number) gives

$$N^{\mu}_{;\mu} = \dot{\upsilon} + \upsilon\theta = 0, \tag{27}$$

where $N^{\mu} = \upsilon u^{\mu}$ is the particle flux, υ is the particle number density and $\theta = u^{\mu}_{;\mu}$ is the expansion of the fluid. From (27), we have

$$\frac{d\upsilon}{\upsilon} = -\theta dt$$

which on integration and using (18) gives the particle number density as

$$\upsilon = \upsilon_0 \exp\left\{\frac{r}{2} Q\right\},$$

where υ_0 is a constant of integration.

At $t=0$, the particle number density is constant and the heat conduction is inversely proportional to the anisotropic direction i.e. r-direction. At large value of t, the particle number density is constant (i.e. $\upsilon = \upsilon_0$) but heat conduction will become disappear.

The expression for the heat conduction [10] is

$$Q_k = K(T_{,i} + T\dot{u}_i)(\delta^i_k - u^i u_k), \tag{28}$$

where K is the thermal conductivity and T is the temperature. Using (28) and zero acceleration of flow vector (i.e. $\dot{u}_i = 0$), we get

$$KT_{,1} = Q, \qquad KT_{,2} = KT_{,3} = KT_{,4} = 0$$

and

$$T = \frac{2e^{-t}}{K} \log r + T_0,$$

where T_0 is a constant. It seems that if the thermal conductivity K increases, the temperature of the universe decreases. Also for infinite time the temperature of the universe becomes constant.

In case (i) (i.e. strange quark matter attached to domain wall), we obtained the negative tension for domain walls (see (23)). In this case we may conclude that domain walls behave like invisible matter due to their negative tension (i.e. negative masses).

In case (ii) (i.e. normal matter attached to the domain wall), when $\gamma = 2$ (i.e. $\rho_m = p_m$), we get the stiff matter domain wall solution (see (24) and (25)). At large value of t, we get negative tension (σ_ω) proportional with cosmological constant (i.e. $\sigma_\omega = -\Lambda$) for domain wall solution.

Finally, we may observed from cases (i) and (ii) that there is a relation between domain walls and cosmological constant (Λ).

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